Binary quantization using Belief Propagation with decimation over factor graphs of LDGM codes

Tomáš Filler, Jessica Fridrich

Binghamton University, USA

BINGHAMTON UNIVERSITY
State University of New York

Overview

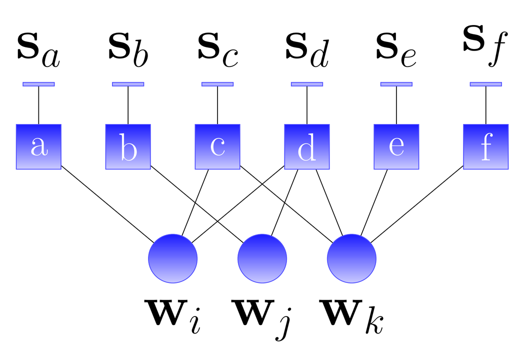
Binary quantization is an important problem for various fields (information hiding, lossy compression). The recent work of Wainwright et al. [1] shows that Low Density Generator Matrix (LDGM) codes combined with Survey Propagation based message-passing algorithm can be used to achieve near-optimal binary quantization in practice.

We propose a much simpler approach based on pure Belief Propagation for quantizing random Bernoulli source with $p = \frac{1}{2}$. This algorithm, which we call Bias Propagation (BiP), achieves the same near-optimal rate-distortion performance and is amenable to theoretical analysis similar to density evolution.

In particular, we are quantizing a random n-bit source sequence $\mathbf{s}$ to the nearest codeword $\mathbf{c_s}$ from an LDGM code $\mathcal{C}$ with rate $R = \frac{m}{n}$. We minimize the average distortion per bit $D = E\big[\frac{1}{n}\sum_{i=1}^{n}|\mathbf{s}_i - (\mathbf{c_s})_i|\big]$.

LDGM code representation

The generator matrix $\mathbf{G} \in \{0,1\}^{n\times m}$ is obtained randomly using degree distributions from edge perspective (ρ, λ).



$\mathbf{s}_f$ ... source bit
f ... check node with degree 1
$\mathbf{w}_k$ ... info bit with degree 4
C ... set of all check nodes
V ... set of all info-bits

$C(i) = \{a \in C \,|\, a \text{ is connected to } i\}$

$V(a) = \{i \in V \,|\, a \text{ is connected to } i\}$ $\quad \overline{V}(a) = V(a) \cup \{\mathbf{s}_a\}$

Bitwise MAP estimation over weighted codewords

Finding $\mathbf{c_s}$ ($\mathbf{c_s} = \mathbf{G}\mathbf{w_s}$, $\mathbf{w_s} \in \{0,1\}^m$) is equivalent to finding MAP assignment in the following probability distribution

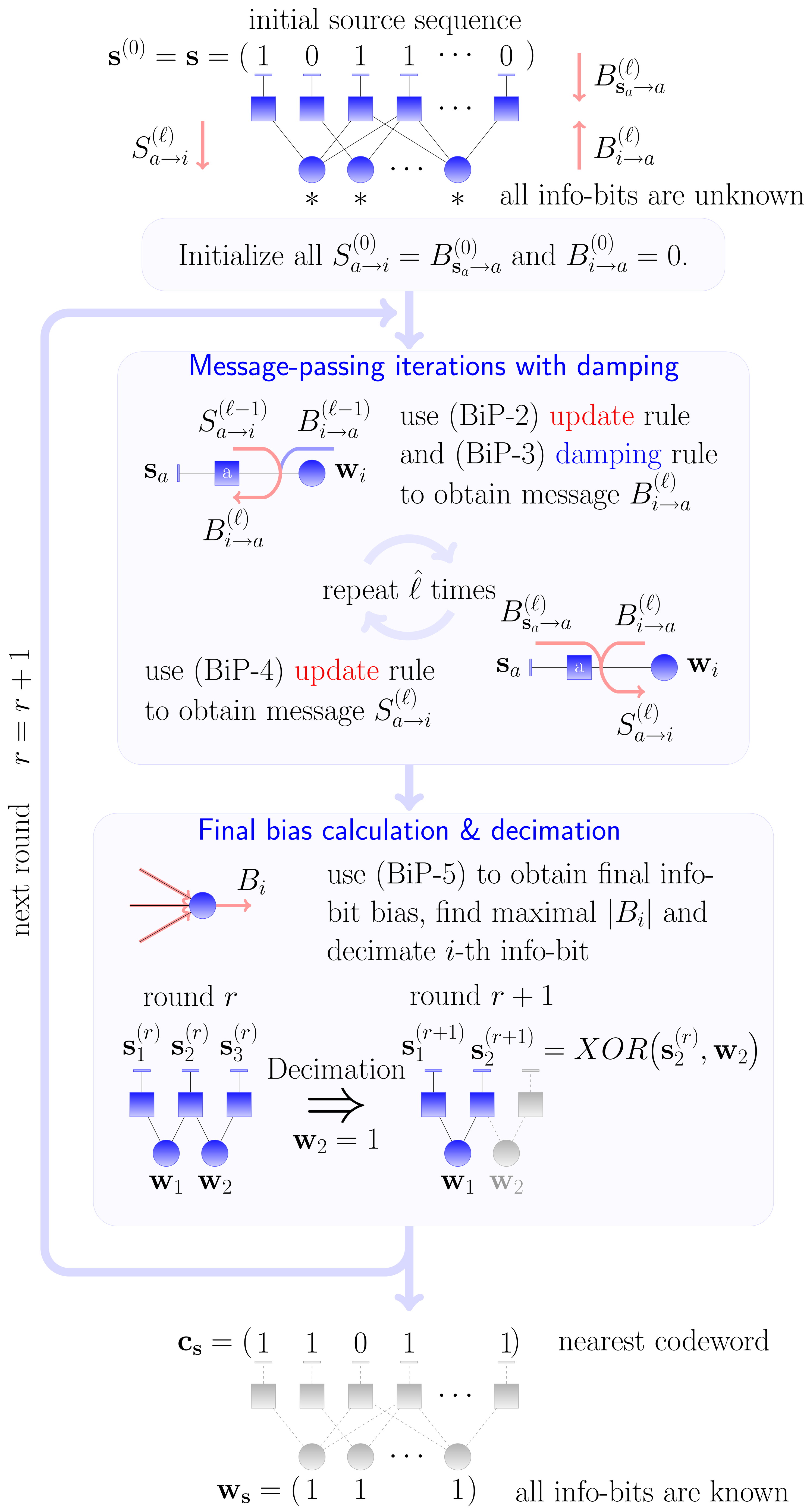
$$P(\mathbf{w}|\mathbf{s};\gamma) = \frac{1}{Z}e^{-2\gamma d_H(\mathbf{Gw},\mathbf{s})},$$

where Z is a normalization constant and $d_H(\mathbf{Gw},\mathbf{s})$ is the Hamming distance between $\mathbf{Gw}$ and $\mathbf{s}$. We find $\mathbf{w_s}$ using bitwise MAP

$$\left(\mathbf{w_s}\right)_i = \arg\max_{\mathbf{w}_i \in \{0,1\}} P(\mathbf{w}_i|\mathbf{s};\gamma).$$

The marginal probabilities $P(\mathbf{w}_i|\mathbf{s};\gamma)$ can be calculated efficiently using Belief Propagation algorithm.

Bias Propagation algorithm (BiP)



BiP update equations

$B^{(\ell)}_{i\to a}$ ... **bias** of info-bit i with respect to check a in ℓ-th iteration
$S^{(\ell)}_{a\to i}$ ... **satisfaction** of check a with respect to i in ℓ-th iteration
$B^{(\ell)}_{\mathbf{s}_a\to a}$ ... **constant source bias** message

$$B^{(\ell)}_{\mathbf{s}_a\to a} = (-1)^{\mathbf{s}_a}\tanh(\gamma) \qquad \text{(BiP-1)}$$

$$\tilde{B}_{i\to a} = \frac{\Pi_{b\in C(i)\setminus\{a\}}\left(1+S^{(\ell-1)}_{b\to i}\right) - \Pi_{b\in C(i)\setminus\{a\}}\left(1-S^{(\ell-1)}_{b\to i}\right)}{\Pi_{b\in C(i)\setminus\{a\}}\left(1+S^{(\ell-1)}_{b\to i}\right) + \Pi_{b\in C(i)\setminus\{a\}}\left(1-S^{(\ell-1)}_{b\to i}\right)} \qquad \text{(BiP-2)}$$

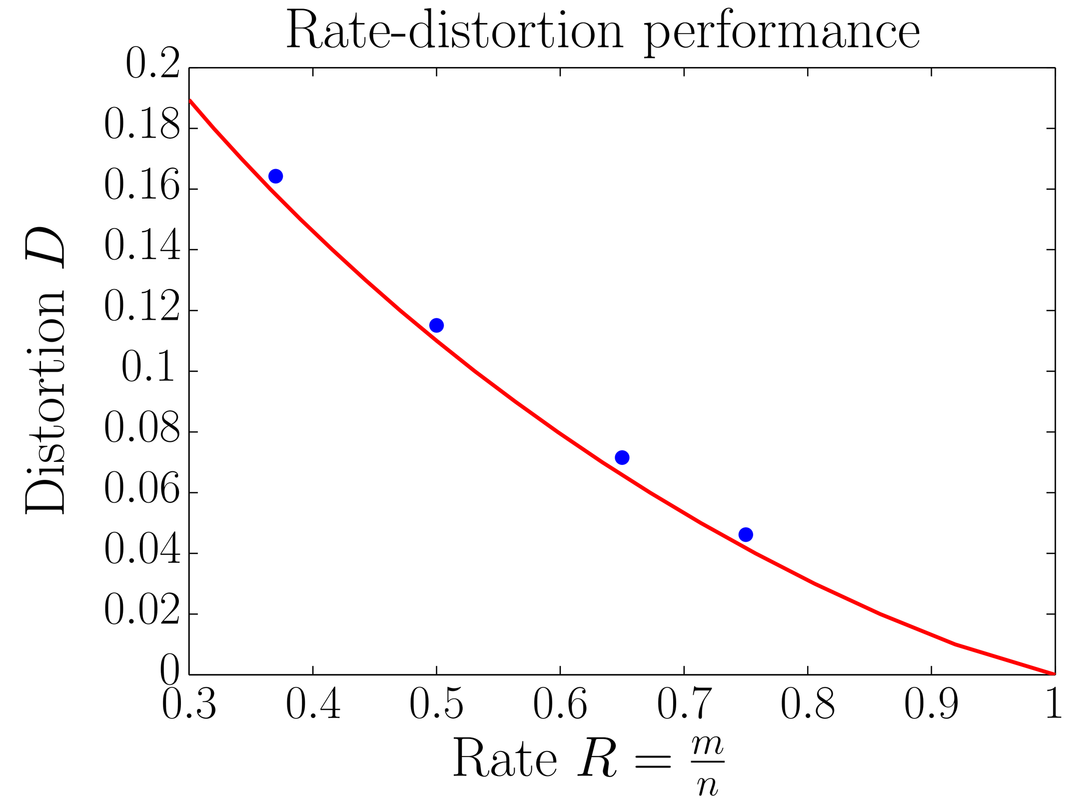
$$B^{(\ell)}_{i\to a} = \frac{\sqrt{(1+\tilde{B}_{i\to a})(1+B^{(\ell-1)}_{i\to a})} - \sqrt{(1-\tilde{B}_{i\to a})(1-B^{(\ell-1)}_{i\to a})}}{\sqrt{(1+\tilde{B}_{i\to a})(1+B^{(\ell-1)}_{i\to a})} + \sqrt{(1-\tilde{B}_{i\to a})(1-B^{(\ell-1)}_{i\to a})}} \qquad \text{(BiP-3)}$$

$$S^{(\ell)}_{a\to i} = \prod_{j\in\overline{V}(a)\setminus\{i\}} B^{(\ell)}_{j\to a} \qquad \text{(BiP-4)}$$

$$B_i = \frac{\Pi_{b\in C(i)}\left(1+S^{(\hat{\ell})}_{b\to i}\right) - \Pi_{b\in C(i)}\left(1-S^{(\hat{\ell})}_{b\to i}\right)}{\Pi_{b\in C(i)}\left(1+S^{(\hat{\ell})}_{b\to i}\right) + \Pi_{b\in C(i)}\left(1-S^{(\hat{\ell})}_{b\to i}\right)} \qquad \text{(BiP-5)}$$

Results

Each • is average over 100 random trials and code length $n = 10^4$. Rate-distortion bound is in the form $R(D) = 1 - H(D)$.



Average throughput (number of source bits quantized per second) was 11kb/sec.

References

[1] M. J. Wainwright and E. Maneva. Lossy source encoding via message-passing and decimation over generalized codewords of LDGM codes. In *Proceedings of the International Symposium on Information Theory, Adelaide, Australia*, September 2005.