EECE 580B Modern Coding Theory

Quick Quiz 1

1. Consider the following code containing two codewords:

$$\{(0\,0\,0\,0\,0\,0\,0\,0\,0\,0), (0\,1\,1\,0\,0\,1\,0\,1\,0\,1)\}$$

- Is this a linear code? YES, it forms a linear subspace
- What is its dimension? k = 1
- What is its minimal distance? d_{min} = 5
- What is its codimension? Codimension is n-k = 9
- Write down its generator matrix. $(0\,1\,1\,0\,0\,1\,0\,1\,0\,1)$

2. Consider the following binary matrix:

$$G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

- What is the dimension of the code whose generator matrix is G?

  Dimension is 2. Last two rows are linearly dependent.

3. How many elements does the set $GF(2)^n$ contain? What are its elements?
   $GF(2)^n$ contains 2^n binary vectors of length n.

4. List all possible codewords from code described by the following parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

   There is only one codeword satisfying $Hx^T = 0$, that is the all-zero codeword.

5. Consider a binary linear [n,k] code.
   - How long are the codewords (how many bits)? n
   - How many codewords does the code have? 2^k
   - How many codewords are in the dual code to this code? 2^{n-k}

6. Let C be linear $[n, k]$ code. What is the rate of the dual code to C? 1-k/n

7. Is the set

$$\left\{ x \in GF(2)^3 \,\middle|\, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} x^T = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

   a linear code? NO, it does not contain the all-zero codeword. This set is called a coset.