EECE 580B
Modern Coding Theory

Low-Density Parity-Check Codes
(1) Graphical representation of codes

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Low-Density Parity-Check Matrix

H =

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1
2	0	0	0	0	0	0	1	1	0	0	1	1	0	1	0	1	0	0	0	0
3	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	1
4	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0
5	1	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0
6	0	0	0	0	0	0	1	0	0	0	1	1	0	1	1	0	0	0	0	1
7	0	0	0	1	1	1	0	1	0	0	0	0	1	0	1	0	0	0	0	0
8	1	0	1	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0
9	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
10	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	1	1	0

6 ones per every row, 3 ones per every column

Graphical Representation of a Linear Code

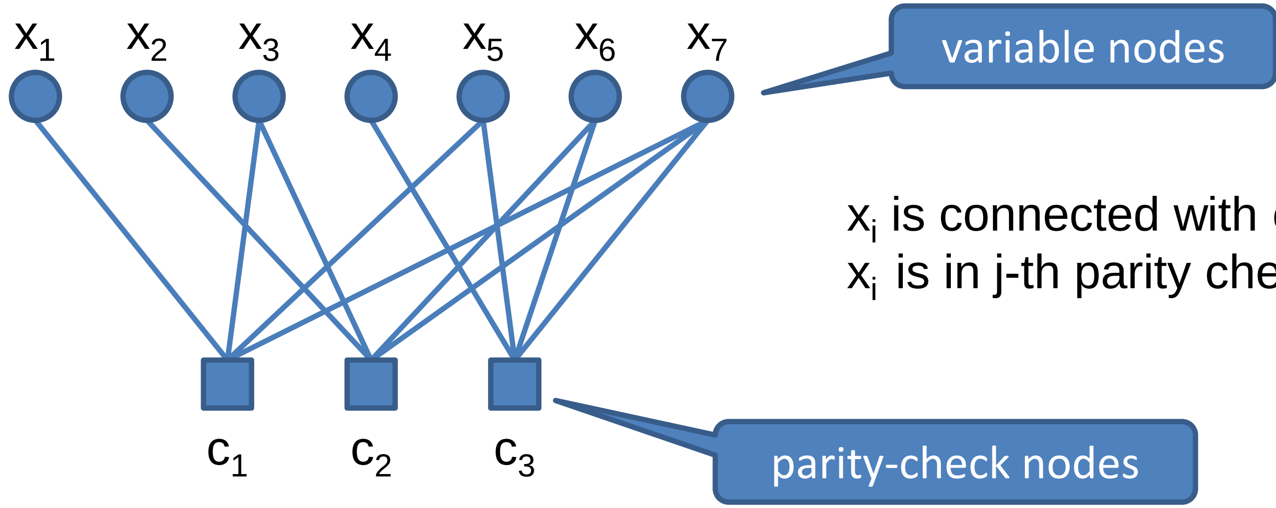
Parity check matrix of [7,4] linear code:

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$c_1 : x_1 + x_3 + x_5 + x_7 = 0$$
$$c_2 : x_2 + x_3 + x_6 + x_7 = 0$$
$$c_3 : x_4 + x_5 + x_6 + x_7 = 0$$

3 parity-check equations $c_1,\ldots,c_3$

Tanner graph:



x_i is connected with c_j iff the variable x_i is in j-th parity check equation.

Can be used with arbitrary parity-check matrix. Hamming code H_3 is used as an illustrative example.

Graphical Representation of a Linear Code (cont.)

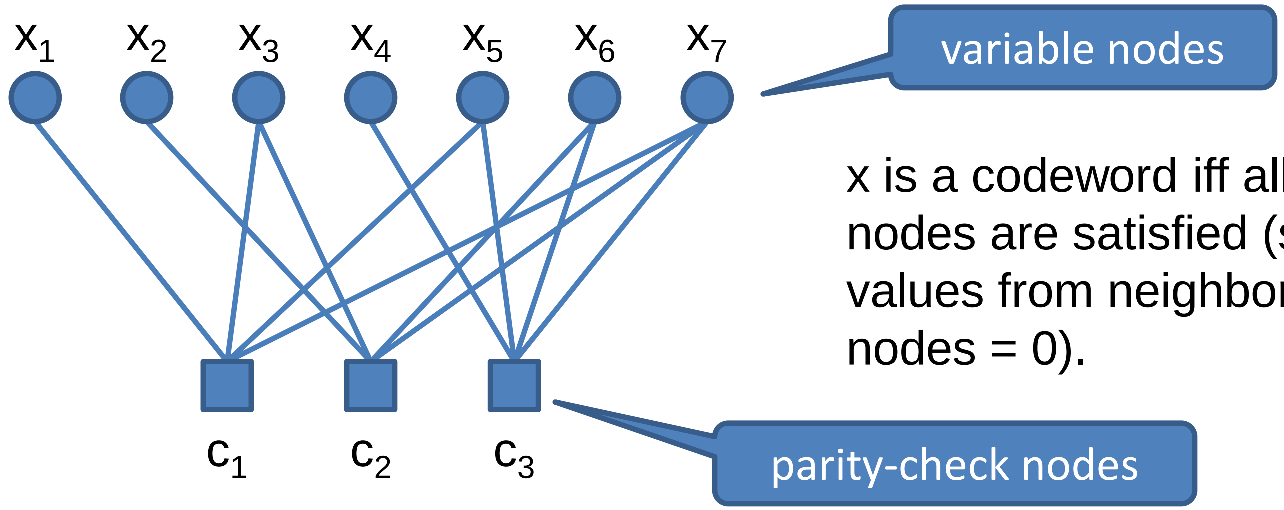
Parity check matrix of [7,4] linear code:

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Code: $C = \{x \in GF(2)^n \mid Hx^T = 0\}$

Tanner graph:

<u>Degree of a node</u> = number of connected edges.



x is a codeword iff all parity-check nodes are satisfied (sum of all values from neighboring variable nodes = 0).

Can be used with arbitrary parity-check matrix. Hamming code H_3 is used as an illustrative example.

Degree Distribution

(from node perspective)

- Notation how to describe Tanner graph of FIXED size.
- Degree distribution $(\Lambda(x), P(x))$

  $\Lambda(x) = \Lambda_1 x + \Lambda_2 x^2 + \ldots + \Lambda_{lmax} x^{lmax}$ $\quad$ Λ_i = # of variable nodes of degree i

  $P(x) = P_1 x + P_2 x^2 + \ldots + P_{rmax} x^{rmax}$ $\quad$ P_i = # of check nodes of degree i

- Example ([7,4] Hamming code):

  $\Lambda(x) = 3x + 3x^2 + 1x^3, \quad P(x) = 3x^4$

- Properties:
  - What is $\Lambda(1)$, $P(1)$?
  - How do you express design rate in terms of $\Lambda(x)$ and $P(x)$?
  - What is $\Lambda'(1)$, $P'(1)$?

Normalized Degree Distribution

(from node perspective)

- Notation how to describe LDPC codes (of arbitrary size).
- Degree distribution $(\Lambda(x), P(x))$

  $L(x) = L_1x+L_2x^2+\ldots+ L_{lmax}x^{lmax}$ — L_i = relative # of var. nodes of deg. i

  $R(x) = R_1x+R_2x^2+\ldots+ R_{rmax}x^{rmax}$ — P_i = relative # of check nodes of degree i

- Example ([7,4] Hamming code):

  $L(x) = 3/7x+3/7x^2+1/7x^3$, $R(x) = x^4$

- Properties:
  - What is $L'(1)$, $R'(1)$?
  - How do you express design rate in terms of $L(x)$ and $R(x)$?