EECE 580B Modern Coding Theory
Final Exam

December 9, 2009

All problems in this final exam are related to the Binary Additive White-Gaussian-Noise Channel (BAWGNC). The notation used here is taken from the handout you received previously. In the whole exam, be careful whether you are working with E_N/σ^2, E_b/N_0, or with E_b/N_0 in decibels.

Problem 1: (Capacity of the BAWGNC) - Weight: 25%

- What is the minimal SNR E_b/N_0 in dB under which a linear code of rate $R = 1/2$ can reduce the bit-error probability P_b to zero when communicating over the BAWGNC?

Hints:

- You need to evaluate the capacity, $C_{BAWGNC}(\sigma)$, of the BAWGNC. Start by plotting the graph of

$$g_{\sigma^2}(y) = -\Phi(y, \sigma^2) \log_2 \Phi(y, \sigma^2)$$

  as a function of y for a fixed value of σ^2. Use the `quad` Matlab command to evaluate the integral.
- Find one value of the noise variance σ^2 for which the capacity of the BAWGNC is in the interval $[0.49, 0.51]$.

As an output of this problem, I want one number (the SNR in dB) from you and all the code used to calculate it.

Problem 2: (Bit-error probability of uncoded transmission) - Weight: 30%

- Plot the bit-error probability P_b vs. E_b/N_0 for the uncoded ($R = 1$) transmission over the BAWGNC. Put $\log_{10} P_b$ on the y axis, and E_b/N_0 in dB on the x axis, respectively. Use the interval $[-10, 10]$ as a range of x axis (E_b/N_0 in dB).

As an output of this problem, I want the graph and all the code used to generate it.

These four degree distributions, $\{L_i(x), R_i(x)\}_{i=1}^4$, will be used in the subsequent experiments. They can be obtained from the course web site (file "dd.mat"). In this file, the degree distributions are stored as cell arrays ($L_i = \texttt{L\{i\}}$ and $R_i = \texttt{R\{i\}}$).

$$\begin{aligned}
L_1(x) &= x^3 && (1)\\
R_1(x) &= x^6, \\
L_2(x) &= 0.58333353703699x^3 + 0.41666646296301x^9 && (2)\\
R_2(x) &= x^{11}, \\
L_3(x) &= 0.48394942887028x^2 + 0.29442753267077x^3 + 0.085134453286822x^6 + 0.074055964589733x^7 && (3)\\
&\quad + 0.062432620582393x^{20} \\
R_3(x) &= 0.74193548387097x^8 + 0.25806451612903x^9, \\
L_4(x) &= 0.9852869602657x^4 + 0.0147130397343x^{19} && (4)\\
R_4(x) &= 0.55860904635731x^8 + 0.44139095364269x^9,
\end{aligned}$$

Use the following Matlab code to implement the BAWGNC with variance $\sigma^2 =$ `sigma2`:

```
function [ y ] = bawgnc_channel( x, sigma2 )
  %BAWGNC_CHANNEL Simulates binary additive white-gaussian noise channel with
  % variance sigma2
  % Input:  x=(x_1,...,x_n)  x_i = 0/1 binary
  % Output: signal y_i = (-1)^x_i + zero-mean Gaussian noise with variance sigma2
  std = sqrt(sigma2);         % calculates the standard deviation
  noise = std*randn(size(x)); % generate zero-mean Gaussian signal
  y = (-1).^x + noise;
end
```

Problem 3: (Bit-error probability of coded transmission) - Weight: 45%

Evaluate the bit-error probability P_b of codes based on each degree distribution mentioned above for the length $n = 1000$ under the BAWGNC with $E_b/N_0 \in [-10, 10]$ dB when decoded with the BP algorithm with the maximum number of 10 iterations. Plot the results into the graph you obtained from Problem 2.

To evaluate one specific code over the BAWGNC with a known SNR (you have a parity-check matrix $\mathbb{H}$ and SNR E_b/N_0 in dB), use the following pseudo-code to obtain P_b

```
for i = 1 to 10 // repeat the same experiment 10 times and set P_b to the mean
  let s = random bits
  let x = encode vector s into a codeword x using the par. check matrix H
  send x over the BAWGNC and obtain y
  let LLR = log-likelihood ratios calculated from y for the BAWGNC with the known SNR
  x_hat = decode y using the BP algorithm with the maximum of 10 iterations
  P_b_vec(i) = average number of errors in INFORMATION BITs of x_hat
end
P_b = mean( P_b_vec )
```

Experiment with the channel parameter E_b/N_0 to plot smooth curves $\log_{10} P_b$ vs. E_b/N_0 in dB (show the results for at least six SNR values with nonzero P_b for each degree distribution). Mark the minimal SNR value obtained from Problem 1 in the same graph. This will show the lower bound on SNR for any code of rate $R = 0.5$.

The purpose of this graph is to show how the bit-error probability P_b can be reduced by using LDPC codes with the BP algorithm when compared with the uncoded transmission.

As an output of this problem, show one graph with five curves (one from Problem 2 and four from this problem) and all the code used to obtain it.