EECE 580B Modern Coding Theory

Final Exam

December 9, 2009

All problems in this final exam are related to the Binary Additive White-Gaussian-Noise Channel (BAWGNC). The notation used here is taken from the handout you received previously. In the whole exam, be careful whether you are working with $E_b/\sigma^2$, $E_b/N_0$, or with $E_b/N_0$ in decibels.

**Problem 1: (Capacity of the BAWGNC) - Weight: 25%**

- What is the minimal SNR $E_b/N_0$ in dB under which a linear code of rate $R = 1/2$ can reduce the bit-error probability $P_b$ to zero when communicating over the BAWGNC?

Hints:
- You need to evaluate the capacity, $C_{BAWGNC}(\sigma)$, of the BAWGNC. Start by plotting the graph of
  $$g_{\sigma^2}(y) = -\Phi(y, \sigma^2) \log_2 \Phi(y, \sigma^2)$$
as a function of $y$ for a fixed value of $\sigma^2$. Use the `quad` Matlab command to evaluate the integral.
- Find one value of the noise variance $\sigma^2$ for which the capacity of the BAWGNC is in the interval $[0.49, 0.51]$. As an output of this problem, I want one number (the SNR in dB) from you and all the code used to calculate it.

**Problem 2: (Bit-error probability of uncoded transmission) - Weight: 30%**

- Plot the bit-error probability $P_b$ vs. $E_b/N_0$ for the uncoded ($R = 1$) transmission over the BAWGNC. Put $\log_{10} P_b$ on the y axis, and $E_b/N_0$ in dB on the x axis, respectively. Use the interval $[-10, 10]$ as a range of x axis ($E_b/N_0$ in dB).

As an output of this problem, I want the graph and all the code used to generate it.

These four degree distributions, $\{L_i(x), R_i(x)\}^4_{i=1}$, will be used in the subsequent experiments. They can be obtained from the course web site (file “dd.mat”). In this file, the degree distributions are stored as cell arrays ($L_i = L\{i\}$ and $R_i = R\{i\}$).

\[
\begin{align*}
L_1(x) &= x^3 \\
R_1(x) &= x^6, \\
L_2(x) &= 0.5833353703699x^3 + 0.4166646296301x^9 \\
R_2(x) &= x^{11}, \\
L_3(x) &= 0.4839428870282 + 0.29442753267077x^3 + 0.085134453286822x^6 + 0.074055964589733x^7 \\
&\quad+ 0.062432620582393x^{20} \\
R_3(x) &= 0.74193548387097x^6 + 0.25806451612903x^9, \\
L_4(x) &= 0.9852869602657x^4 + 0.0147130397343x^{19} \\
R_4(x) &= 0.55860904635731x^8 + 0.44139095364269x^9,
\end{align*}
\]
Use the following Matlab code to implement the BAWGNC with variance $\sigma^2 = \text{sigma2}$:

```matlab
function [ y ] = bawgnc_channel( x, sigma2 )
    %BAWGNC_CHANNEL Simulates binary additive white-gaussian noise channel with
    % variance sigma2
    % Input: x=(x_1,...,x_n) x_i = 0/1 binary
    % Output: signal y_i = (-1)\textasciicircum{}x_i + zero-mean Gaussian noise with variance sigma2
    std = sqrt(sigma2); % calculates the standard deviation
    noise = std*randn(size(x)); % generate zero-mean Gaussian signal
    y = (-1).\textasciicircum{}x + noise;
end
```

**Problem 3: (Bit-error probability of coded transmission) - Weight: 45%**

Evaluate the bit-error probability $P_b$ of codes based on each degree distribution mentioned above for the length $n = 1000$ under the BAWGNC with $E_b/N_0 \in [-10, 10]$ dB when decoded with the BP algorithm with the maximum number of 10 iterations. Plot the results into the graph you obtained from Problem 2.

To evaluate one specific code over the BAWGNC with a known SNR (you have a parity-check matrix $H$ and SNR $E_b/N_0$ in dB), use the following pseudo-code to obtain $P_b$

```matlab
for i = 1 to 10 // repeat the same experiment 10 times and set P_b to the mean
    let s = random bits
    let x = encode vector s into a codeword using the par. check matrix H
    send x over the BAWGNC and obtain y
    let LLR = log-likelihood ratios calculated from y for the BAWGNC with the known SNR
    x_hat = decode y using the BP algorithm with the maximum of 10 iterations
    P_b_vec(i) = average number of errors in INFORMATION BITS of x_hat
end
P_b = mean( P_b_vec )
```

Experiment with the channel parameter $E_b/N_0$ to plot smooth curves $\log_{10} P_b$ vs. $E_b/N_0$ in dB (show the results for at least six SNR values with nonzero $P_b$ for each degree distribution). Mark the minimal SNR value obtained from Problem 1 in the same graph. This will show the lower bound on SNR for any code of rate $R = 0.5$.

The purpose of this graph is to show how the bit-error probability $P_b$ can be reduced by using LDPC codes with the BP algorithm when compared with the uncoded transmission.

As an output of this problem, show one graph with five curves (one from Problem 2 and four from this problem) and all the code used to obtain it.