EECE 580B Modern Coding Theory | Homework assignment #4
WWW: http://dde.binghamton.edu/filler/mct/hw/4 | Due date: October 22, 2:50pm

LDPC code construction

Problem 1 (Tanner graph of a linear code):
Let C be a linear code described by the following parity-check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Draw a Tanner graph corresponding to the parity-check matrix H.

Problem 2:
Let $(L(x), R(x))$ be a normalized degree distribution (from node perspective) defined as

$$L(x) = \sum_{i=1}^{l_{max}} L_i x^i \quad and \quad R(x) = \sum_{i=1}^{r_{max}} R_i x^i,$$

where L_i denotes the relative number of variable nodes of degree i, and R_i denotes the relative number of parity-check nodes of degree i. The design rate $r = 1 - L'(1)/R'(1)$, where $L'(1)$ is the derivative of $L(x)$ at 1. Write an algorithm that, for given polynomials $L(x), R(x)$, and code length n, creates random parity-check matrix H of approximate size $(1-r)n \times n$ with the relative number of columns (rows) having i ones being close to L_i (R_i). I expect the Matlab function in the following form:

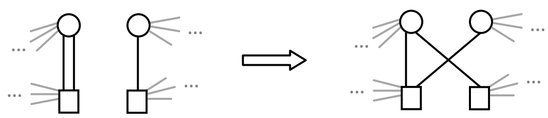
```
function [H rate] = ldpc_create_matrix(L, R, n)
% L and R describe the normalized degree distribution
%   (3,6) regular LDPC code will be described by L=[0 0 1] and R=[0 0 0 0 0 1]
% return sparse parity-check matrix H, rate of the code
end
```

and all graphs and output you obtain from the following test script (see test_ldpc_create.m on the web):

```
clc; clear;
n = 1000;

% (3,6) regular LDPC code
L = [0 0 1];
R = [0 0 0 0 0 1];
[H rate] = ldpc_create_matrix(L, R, n);
figure; spy(H);
figure;subplot(2,1,1); hist(full(sum(H,1))); subplot(2,1,2); hist(full(sum(H,2)));
disp(rate);

% irregular LDPC code 1
L = [0 0.4994 0.3658 0 0 0 0.0581 0 0 0 0 0 0.0767];
R = [0 0 0 0 0 0 1];
[H rate] = ldpc_create_matrix(L, R, n);
figure; spy(H);
figure;subplot(2,1,1); hist(full(sum(H,1))); subplot(2,1,2); hist(full(sum(H,2)));
disp(rate);

% irregular LDPC code 2
L = [0 0 0.76 0 0 0 0 0.24];
R = [0 0 0 0 0 1];
[H rate] = ldpc_create_matrix(L, R, n);
```

```
figure; spy(H);
figure;subplot(2,1,1); hist(full(sum(H,1))); subplot(2,1,2); hist(full(sum(H,2)));
disp(rate);
```

The algorithm for creating random matrix (or a Tanner graph) with specified number of ones in rows and columns can be described as follows. Given the normalized degree distribution pair $(L(x), R(x))$, create the (non-normalized) degree distribution polynomials $(\Lambda(x), P(x))$, as $\Lambda(x) = nL(x)$ and $P(x) = (1-r)nR(x)$. Round the coefficients of $\Lambda(x)$ and $P(x)$ to integers. Make sure that the number of edges going from the variable nodes, $\Lambda'(1)$, is the same as the number of edges going from parity-check nodes, $P'(1)$. If not, then adjust the degree of sufficient number of parity-check nodes by one starting from the parity-check nodes with the highest degree and update $P(x)$. Make sure that $P'(1) = \Lambda'(1)$. Now imagine that every variable node of degree i contain i sockets from which i edges will be connected to some parity-check nodes. The same holds for the parity-check nodes thus there are $P'(1)$ sockets at each side. Label each sockets by numbers from 1 to $P'(1)$ and let σ be a random permutation over the set $\{1, \dots, P'(1)\}$. The Tanner graph is constructed by connecting i-th socket on the variable side with $\sigma(i)$-th socket on the parity-check side. This approach may create a small number of double edges which have to be resolved as shown in the following example. The result is a random Tanner graph (or parity-check matrix).



Hints, warnings and suggestions:

- Try to work with a (3,6) regular code of length 10 first.
- Try to work with sparse matrices. See 'doc sparse' to know how to create sparse matrix. Try and study the matrix produced by the following code:

```
H = full(sparse([1 1 2 1],[1 3 5 1],ones(1,4)))
```

Optional problem (degree distribution from edge perspective): (will be graded softly)
Let $(L(x), R(x))$ be a normalized degree distribution (from node perspective) as defined above. Suppose that the Tanner graph is obtained randomly with the variable and parity-check degrees following $L(x), R(x)$. Define λ_i as the fraction of edges connected to variable nodes of degree i. Similarly, ρ_i as the fraction of edges connected to parity-check nodes of degree i. Define <u>degree distribution from edge perspective</u> as the following polynomials (the term x^{i-1} is intentional)

$$\lambda(x) = \sum_{i=1}^{l_{max}} \lambda_i x^{i-1} \quad and \quad \rho(x) = \sum_{i=1}^{r_{max}} \rho_i x^{i-1}.$$

Hint: What is $\int_0^x \lambda(z)dz$?

- Give a simple formula for calculating $\lambda(x)$ and $\rho(x)$ from $L(x)$ and $R(x)$.
- Give a simple formula for calculating $L(x)$ and $R(x)$ from $\lambda(x)$ and $\rho(x)$.
- Express the design rate in the terms of $\lambda(x)$ and $\rho(x)$.
- Express the average variable node degree and average parity-check node degree in terms of $\lambda(x)$ and $\rho(x)$.