EECE 580B Modern Coding Theory
WWW: http://dde.binghamton.edu/filler/mct/hw/2

Homework assignment #2
Due date: September 24, 2:50pm

Block error rate

Alternative decoding of the repetition code R_9:

The repetition code R_n consists of two n-bit codewords $R_n = \{00\ldots0, 11\ldots1\}$. Consider the following decoders for the code R_9 over the BSC:

- ML decoder: given channel output $y \in \{0,1\}^n$, the decoder outputs the all-zero codeword if the number of ones in y is less than $\lceil n/2 \rceil$, otherwise it returns the all-one codeword.
- Recursive decoder: set $y_1^3 = (y_1, y_2, y_3)$, $y_4^6 = (y_4, y_5, y_6)$, $y_7^9 = (y_7, y_8, y_9)$. Use the ML decoder for R_3 on every triple y_1^3, y_4^6, y_7^9 to obtain the following bits $\hat{y} = (\hat{y}_1, \hat{y}_2, \hat{y}_3)$. Finally, decode $\hat{y}$ by using the ML decoder for R_3.
  Example: $y = (100\ 111\ 001)$, use the ML decoder for R_3, $\hat{y} = (010)$. The output codeword is $x_{out} = (000000000)$.

Problems (Is recursive decoder optimal for R_9 over BSC?):

1. Suppose you send the all-zero codeword. Find a noisy word $y \in \{0,1\}^9$ that can be corrected by the ML decoder but not by the recursive decoder. Calculate the probability of receiving your word y if the channel is $BSC(f)$. Find the realization of a noisy word y with the largest probability of occurrence.
2. OPTIONAL - Under the assumption that both codewords are sent equally likely, write down the probability of block error for both the ML and the Recursive decoder when the $BSC(f)$ is used. To accept this as optional problem you have to show both probabilities. See course website for description how the optional problems are handled.

Rate vs. block error probability graph (create a graph similar to Figure 1.12 in MacKay):

1. (n,k,d_{min}) code (code of length n, size 2^k with minimum distance d_{min}) can correct up to $\left\lfloor \frac{d_{min}-1}{2} \right\rfloor$ errors introduced by $BSC(f)$. Create a Matlab function that, given the parameters n, k, d_{min}, f, calculates the rate of the code and the probability of block error caused by the ML decoder.

```
function [ R PB ] = code_block_error_BSC(n, k, d_min, f)
   % Implement the body of this function.
   % R  ... rate of the code
   % PB ... probability of block error caused by ML decoder.
end
```

2. (n,k,d_{min}) code can correct up to $\lfloor d_{min} - 1 \rfloor$ erasures introduced by $BEC(e)$. Create a Matlab function that, given the parameters n, k, d_{min}, e, calculates the rate of the code and the probability of block error caused by the ML decoder.

```
function [ R PB ] = code_block_error_BEC(n, k, d_min, e)
   % Implement the body of this function.
   % R  ... rate of the code
   % PB ... probability of block error caused by ML decoder.
end
```

3. The repetition code R_n is a class of (n,1,n) codes, $n \geq 1$. Draw the rate (x axis) vs. the probability of block error (y axis) of repetition codes for $BSC(0.11)$ for $n \in \{1, \ldots, 61\}$, calculate the capacity of this channel and plot it in the graph. Use logarithmic axes for the probability of block error.

4. Draw the rate vs. the probability of block error of repetition codes for $BEC(0.5)$ for $n \in \{1, \dots, 61\}$, calculate the capacity of this channel and plot it in the graph. Use logarithmic axis for the probability of block error.

Hints, warnings and suggestions:

- Use nchoosek(n,k) to calculate the binomial coefficient $\binom{n}{k}$. Do not use factorials to calculate it. You may consider some warnings from Matlab when calculating $\binom{n}{k}$ for larger coefficients n and k. You may use the following commands to disable the warnings for a while:

```
warning off MATLAB:nchoosek:LargeCoefficient
% calculate nchoose k for large coefficients
warning on MATLAB:nchoosek:LargeCoefficient
```

- Learn how to plot graphs in Matlab. See the documentation of the following commands: plot, semilogy, xlabel, ylabel, title.

Purpose of this assignment:

Create a graph showing the performance of known codes. We will use this graph later, when we come up with better codes.